Optimizing Packaging Material Using Calculus

Rationale and Objective

Humans have resorted to manufacturing and processing in the contemporary world for profit, and they produce a great deal, the majority of which is disposed of improperly after usage. This element caused me to become worried about the repercussions of pollution on the environment, which are felt in many ways, including health problems in human life. The media is inundated with new reports almost daily concerning plastics wreaking havoc in the oceans and bio-accumulating in higher-order animals (Rhodes 207). This revelation gave me the idea to research to lower the sector's plastic use. I designed a 1-liter plastic bottle container using the least amount of plastic possible. Ultimately, I decided to settle with a 1-liter capacity since it produced the most. Therefore, this exploration uses calculus to optimize the raw materials required to produce plastic containers to eliminate waste.

Mathematical Computation

To create the optimal surface area of the various shapes that will be considered, I will first determine the surface area of a raw material that will be utilized as a foundation for the calculation in this study. Additionally, since reducing plastic waste entails maximizing raw material area, I will consider surface area rather than volume. This approach is crucial, as maximizing surface area allows for better exposure to natural processes like degradation and decomposition. Furthermore, it is simpler to discover techniques for decreasing plastic waste when the surface area is prioritized over volume, such as creating goods with complex, linked designs that optimize material consumption and reduce waste. Ultimately, this surface-area-centric strategy will help achieve the overarching objectives of lowering plastic waste and advancing sustainability. So, let's examine the diagram down below.

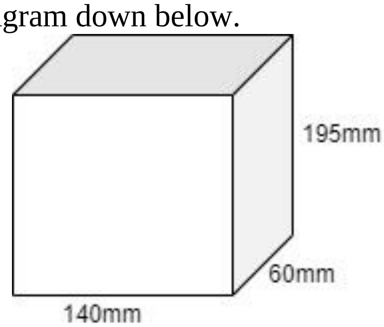


Diagram 1: An example of a 1-litre cuboid

Since this box will be the starting point for my argument, I'll start by figuring out how much surface area (SA) is available to generate a one-liter container. We can determine how much plastic is needed to build the box by calculating its surface area. This data will be essential for assessing possible waste and investigating strategies for material consumption optimization. Furthermore, by calculating the surface area, we can determine if it would be possible to add complex, linked patterns to the design, which would help to reduce plastic waste and advance sustainability. Therefore, by examining the SA, we can take concrete steps towards creating a 1-liter container with minimal environmental impact.

$SA = 2(lw + lh + wh)$s

$$SA = 2((140 \times 60) + (60 \times 195) + (140 \times 195)) = 94800mm^2$$

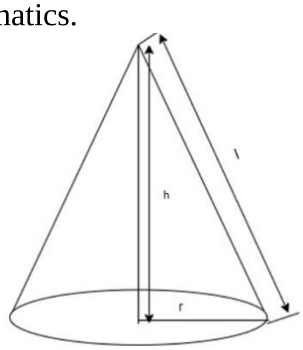
As calculated above, the surface area of the cuboid in diagram 1 is $94800mm^2$. This calculated area will act as the comparison basis of the optimized material.

Conical Shaped Containers

Conical-shaped containers aid in the packaging of goods like ice cream. Below is an image of an actual ice cream cone (diagram 2). This ice cream cone is made with a crispy wafer cone shaped like a cone with a pointed top. The conical shape allows for easy stacking of the ice cream cones, making it convenient for packaging and transportation. In addition, the cone offers a stable foundation for the ice cream scoop, preventing it from toppling over while the customer is enjoying it.

Diagram 2: A real-life example of a conical-shaped container

Consider the conical figure below to understand the connection between this container's shape and mathematics.



$$C_{SA} = \pi r l + \pi r^2 \ldots\ldots (1)$$

$$C_V = \frac{1}{3}\pi r^2 h \ldots. (2)$$

In this case, let us use equation 2 above to make h the subject of the formula as follows

$$3(C_V) = \left(\frac{1}{\cancel{3}}\pi r^2 h\right)\cancel{3}$$

$$3C_V = \pi r^2 h$$

$$\frac{3C_V}{\pi r^2} = \frac{\cancel{\pi r^2}h}{\cancel{\pi r^2}}$$

$$\frac{3C_V}{\pi r^2} = h$$

Nonetheless, according to Pythagoras' theorem (Britannica),

$$l^2 + h^2 = s^2$$

Where $l, h, and\ s\ represent\ the\ length, height, and\ slant\ height\ of\ a\ right-angled\ triangle$. Therefore, to get the length of the triangle, we rearrange the Pythagorean equation as follows.

$$l = \sqrt{r^2 + h^2} \ldots \ldots (3)$$

We get the following equation by substituting this length to the surface area equation.

$$C_{SA} = \pi r \left(\sqrt{r^2 + h^2}\right) + \pi r^2$$

In this case, the above-conducted substitution is crucial in this computation since the surface area of a whole cone must be represented in terms of its radius (r) and vertical height (h). Nevertheless, up to this point, the volume equation below represents the vertical height.

$$\frac{3C_V}{\pi r^2} = h$$

Therefore, this calls for further substitution in the area equation, resulting in the following.

$$C_{SA} = \pi r \left(\sqrt{r^2 + \left(\frac{3C_V}{\pi r^2}\right)^2}\right) + \pi r^2$$

Now, on simplification, we get the following.

$$C_{SA} = \pi r \left(\sqrt{\frac{\pi^2 r^6 + 9{C_V}^2}{\pi^2 r^4}}\right) + \pi r^2$$

This expression can further be simplified by employing the quotient rule below ("Exponents and Square Roots - GRE (Video Lessons, Examples and Solutions)")

$$\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$$

Therefore,

$$C_{SA} = \frac{\sqrt{\pi^2 r^6 + 9{C_V}^2} + \pi r^3}{r} \ldots \ldots (4)$$

At this point, I will calculate the optimum measurements for a conical-shaped packing container that will reduce the required production material, using equation (4) above. As a result, having a solid knowledge of differential calculus will help with the optimization process. Below is how I will apply the differential equation.

$$\frac{dC_{SA}}{dr} = \frac{d}{dr}\left(\frac{\sqrt{\pi^2 r^6 + 9{C_V}^2} + \pi r^3}{r}\right)$$

Nevertheless, I will utilize the quotient rule, which is expressed as follows in differentiation, due to the equation needed to get the derivative of a quotient function ("Quotient Rule For Calculus (w/ Step-by-Step Examples!)"):

$$\frac{dm}{dn} = \frac{(ndm - mdn)}{n^2}$$

Where

$$\frac{dm}{dn} = \frac{dC_{SA}}{dr}$$

So, let m be the numerator and n be the denominator of equation (4) above. Then,

$$m = \sqrt{\pi^2 r^6 + 9{C_V}^2} + \pi r^3$$

And

$$n = r$$

Hence,

$$\frac{dm}{dr} = \frac{d}{dr}\left(\sqrt{\pi^2 r^6 + 9{C_V}^2} + \pi r^3\right)$$

$$\frac{dm}{dr} = \frac{d}{dr}\left(\sqrt{\pi^2 r^6 + 9{C_V}^2}\right) + \frac{d}{dr}\pi r^3$$

$$\frac{d}{dr}\pi r^3 = 3\pi r^2$$

By chain rule knowledge (*Derivatives-Chain rule and power rule PDF*);

$$\frac{d}{dr}\left(\sqrt{\pi^2 r^6 + 9{C_V}^2}\right) = \frac{3\pi^2 r^5}{\sqrt{\pi^2 r^6 + 9{C_V}^2}}$$

$$\frac{dm}{dr} = \frac{3\pi^2 r^5}{\sqrt{\pi^2 r^6 + 9{C_V}^2}} + 3\pi r^2$$

Remember that

$$n = r$$

Therefore,

$$\frac{dn}{dr} = 1$$

We have the complete derivative from the quotient formula stated earlier, as shown below.

$$\frac{dC_{SA}}{dr} = \frac{dm}{dn} = \frac{(ndm - mdn)}{n^2}$$

Therefore,

$$\frac{dC_{SA}}{dr} = \left(r \times \left(\frac{3\pi^2 r^5}{\sqrt{\pi^2 r^6 + 9{C_V}^2}} + 3\pi r^2\right)\right) - \left(1 \times \left(\sqrt{\pi^2 r^6 + 9{C_V}^2} + \pi r^3\right)\right)$$

$$\frac{dC_{SA}}{dr} = \frac{2\pi^2 r^6 + 2\pi r^3\left(\sqrt{\pi^2 r^6 + 9{C_V}^2}\right) - 9{C_V}^2}{r^2\sqrt{\pi^2 r^6 + 9{C_V}^2}} \ldots\ldots\ldots.(5)$$

At this point, the solutions to equation 5 are obtained from the first derivative of equation 4, and it is this outcome that will make obtaining the ideal dimension easier. I will equate it to zero to move on from equation 5 and provide the solution below.

$$\frac{2\pi^2 r^6 + 2\pi r^3 \left(\sqrt{\pi^2 r^6 + 9{C_V}^2}\right) - 9{C_V}^2}{r^2\sqrt{\pi^2 r^6 + 9{C_V}^2}} = 0$$

$$r^2\sqrt{\pi^2 r^6 + 9{C_V}^2}\left(\frac{2\pi^2 r^6 + 2\pi r^3 \left(\sqrt{\pi^2 r^6 + 9{C_V}^2}\right) - 9{C_V}^2}{r^2\sqrt{\pi^2 r^6 + 9{C_V}^2}}\right) = 0$$

$$2\pi^2 r^6 + 2\pi r^3 \left(\sqrt{\pi^2 r^6 + 9{C_V}^2}\right) - 9{C_V}^2 = 0$$

Simplifying the above expression results in the following

$$C_V = \sqrt{\frac{8\pi^2 r^6}{9}}$$

At $\frac{dC_{SA}}{dr} = 0$, C_V is optimum. Thus,

$$C_V = Opt_V = \sqrt{\frac{8\pi^2 r^6}{9}} \ldots\ldots\ldots (6)$$

The optimum volume of a cylindrical, conical-shaped item is represented by equation (6), computed with optimal radius. What, then, is the relationship between the radius and height of an optimum conical object? I'll compare the optimized volume equations with the general cone volume equations to find a solution, as seen below.

$$C_V = Opt_V$$

Thus,

$$\sqrt{\frac{8\pi^2 r^6}{9}} = \frac{1}{3}\pi r^2 h$$

$$\frac{2\pi}{3}\sqrt{2(r^2)^3} = \frac{1}{3}\pi r^2 h$$

$$\frac{\cancel{3}}{\cancel{\pi r^2}}\left(\frac{2\cancel{\pi r^3}\sqrt{2}}{\cancel{3}}\right) = (\frac{1}{\cancel{3}}\cancel{\pi r^2}h) \times \frac{\cancel{3}}{\cancel{\pi r^2}}$$

$$2r\sqrt{2} = h$$

In summary, we may deduce that given a variable factor of 2√2, the height of an ideal cone is precisely proportional to its optimum radius. From another perspective, the following steps would be taken to get the perfect measurements for a conical-shaped object holding one liter of fluid.

$$Opt_V = \sqrt{\frac{8\pi^2 r^6}{9}}$$

$$1000 = \sqrt{\frac{8\pi^2 r^6}{9}}$$

$$100000^2 = \left(\sqrt{\frac{8\pi^2 r^6}{9}}\right)^2 \frac{90000000000}{8\pi^2} = r^6$$

$$r = \sqrt[6]{\frac{90000000000}{8\pi^2}} = 32.3159707$$

$$r \approx 32.32mm$$

After determining the value of (r), let's use the procedure below to determine the optimal height.

$$2r\sqrt{2} = h$$

$$h = \sqrt[6]{\frac{90000000000}{8\pi^2}} \times 2\sqrt{2} = \ 91.41562994h \approx 91.42$$

From the above calculation, the approximate dimensions that could generate the optimum materials are $h \approx 91.42\ mm\ and\ r \approx 32.32\ mm$. Therefore, the optimum surface area of the bottle calculated using these measurements is as follows.

$$C_{SA} = \pi rl + \pi r^2$$

Nevertheless, I discovered earlier that $l = \sqrt{r^2 + h^2}$

Thus,

$$C_{SA} = \pi \times 32.32 \times \sqrt{91.42^{\ 2} + 32.32^2} + \pi \times 32.32^2$$

$$C_{SA} \approx 13132.39mm^2$$

Optimized Material

Following optimization, the approximate area of materials needed to assemble a standard ice cream cone with a minimum capacity of one liter is determined (using optimized measurements). Comparing it to the initial area of the raw material, we find that the $13132.39mm^2 < 94800mm^2$. Calculate the percentage change between the two areas using the formula below ("Percentage Change - Definition, Formula, Examples").

$$\% \, change = \frac{94800mm^2 - 13132.39mm^2}{94800mm^2} \times 100\% \, change \approx 35.04\%$$

According to the computation above, improving a conical container—like a normal ice cream cone—would result in material and cost savings of around 35.04%. Nevertheless, in real life, this value may be wrong, especially considering that in Diagram 2, the measurements of the indicated conical material are not regular and overlap each other, meaning that it might even be more prominent to hold small quantities.

Cylindrical Containers

A cylinder is an object composed of two circles that are comparable to one another on parallel planes, their interiors, and any line segments that cross each other and terminate in circular regions parallel to the segment that includes the centers of the two circles ("Cylinder (Shape, Properties, Formulas, Surface Area, Volume, Examples)"). Since the radius of a base in this example determines the diameter of a cylinder, the circles and their interiors form the basis for the structure. The cylinder's width and height equal its two bases' lengths and altitude. Its altitude is a segment that runs perpendicular to the planes of the two bases. The actual 1-liter cylindrical bottle shown below is an example.

Diagram 3: An example of a cylindrical container

Optimum Surface Area

The volume (Cy_V) and the cylinder's surface area (Cy_{SA}) can be calculated using the formulas below.

$$Cy_V = \pi r^2 h \dots \dots . i \quad Cy_{SA} = 2\pi r^2 + 2\pi rh \dots . . ii$$

I will begin this section by rearranging the volume equation such that the cylinder's height (h) is the focus of the subsequent expression. The object's height is determined by;

$$\frac{Cy_V}{\pi r^2} = \frac{\cancel{\pi r^2} h}{\cancel{\pi r^2}}$$

Therefore,

$$h = \frac{Cy_V}{\pi r^2} \ldots\ldots iii$$

Substituting equation iii in ii we get;

$$Cy_{SA} = 2\pi r^2 + 2\pi r h$$

$$Cy_{SA} = 2\pi r^2 + 2\pi r \left(\frac{Cy_V}{\pi r^2}\right)$$

The equation is prepared for differential calculus, as seen below.

$$\frac{dCy_{SA}}{dr} = \frac{d}{dr}\left(2\pi r^2 + 2\pi r \left(\frac{Cy_V}{\pi r^2}\right)\right)$$

$$\frac{dCy_{SA}}{dr} = \frac{d}{dr}(2\pi r^2) + \frac{d}{dr}\left(2\pi r \left(\frac{Cy_V}{\pi r^2}\right)\right)$$

$$\frac{dCy_{SA}}{dr} = 4\pi r + \frac{d}{dr}\left(\left(\frac{2Cy_V}{r}\right)\right)$$

$$\frac{d}{dr}\left(\left(\frac{2Cy_V}{r}\right)\right) = \frac{d}{dr}(2Cy_V r^{-1})$$

$$\frac{d}{dr}(2Cy_V r^{-1}) = -2Cy_V r^{-2} = -\frac{2Cy_V}{r^2}$$

Thus

$$\frac{dCy_{SA}}{dr} = 4\pi r - \frac{2Cy_V}{r^2}$$

I will equate the above equation to zero and solve it as follows to achieve the optimum results possible;

$$\left(4\pi r - \frac{2Cy_V}{r^2}\right) r^2 = 0$$

$$4\pi r^3 - 2Cy_V = 0$$

$$\frac{4\pi r^3}{\cancel{2}} = \frac{\cancel{2}Cy_V}{\cancel{2}}$$

$$Cy_V = 2\pi r^3$$

Recall that at $\frac{dCy_{SA}}{dr} = 0$, $Cy_V = Opt_V$. Hence,

$$Opt_V = 2\pi r^3 \ldots\ldots d$$

Equation d above provides the method for figuring out a cylindrical container's maximum volume, given its shape. The formula describes the volume in terms of its ideal radius, but it does not offer the dimensions needed to optimize the material to construct the optimal packing for cylindrical-shaped containers. I will start by computing a second derivative from the following equation to ascertain if the radius of the volume is maximal or minimal and whether it is ideal ("How to Find Maximum and Minimum Points Using Differentiation").

$$\frac{dCy_{SA}}{dr} = 4\pi r - \frac{2Cy_V}{r^2}$$

$$d^2 Cy_{SA} = \left(4\pi r - \frac{2Cy_V}{r^2}\right) dr^2$$

$$\frac{d^2Cy_{SA}}{dr^2} = \frac{d^2}{dr^2}(4\pi r) - \frac{d^2}{dr^2}\left(\frac{2Cy_V}{r^2}\right)$$

Thus,

$$\frac{d^2Cy_{SA}}{dr^2} = 4\pi - \frac{d^2}{dr^2}\left(\frac{2Cy_V}{r^2}\right)$$
$$\frac{d^2}{dr^2}\left(\frac{2Cy_V}{r^2}\right) = \frac{d^2}{dr^2}(2Cy_V r^{-2})$$
$$\frac{d^2}{dr^2}(2Cy_V r^{-2}) = -4Cy_V r^{-3}$$
$$\frac{d^2Cy_{SA}}{dr^2} = 4\pi - (-4Cy_V r^{-3})$$
$$\frac{d^2Cy_{SA}}{dr^2} = 4\pi + 4Cy_V r^{-3} = 4\pi + \frac{4Cy_V}{r^3}$$

Recall that the optimum volume (Opt_V) is given as

$$Opt_V = 2\pi r^3$$

Therefore,

$$\frac{d^2Cy_{SA}}{dr^2} = 4\pi + 8\pi = 12\pi$$

Comparing 12π to zero, we observed that 12π is greater than zero ($12\pi > 0$). Let us examine the connection between the cylinder's radius and height. The volume of an optimized cylindrical item and a generic cylindrical thing will be compared, as seen in the image below. The cylinder's height dramatically influences how much volume it contains. Generally speaking, the volume of the cylinder will grow if the height is increased while the radius remains the same. Conversely, the volume will decrease if the height is lowered while maintaining the same radius. Consequently, making the most of a cylindrical object's height might result in a more effective use of available space.

$$Opt_V = Cy_V$$
$$\frac{2\cancel{\pi r^3}}{\cancel{\pi r^2}} = \frac{\cancel{\pi r^2}h}{\cancel{\pi r^2}}$$
$$2r = h \dots.e$$

As a result, the height given in equation e is also optimum since the radius is. This fact leads one to the conclusion that a cylindrical container's height needs to match either two radii or the diameter of the cylindrical container in question for it to have the smallest surface area. Thus, the following steps could be taken if someone needs to find out what kind of material would be needed to pack a 1-liter liquid:

$$2\pi r^3 = 100000mm^3 r^3 = \frac{100000m^3}{2\pi} = \frac{50000mm^3}{\pi}$$
$$r = \sqrt[3]{\frac{50000mm^3}{\pi}} = 25.15060604$$
$$r \approx 25.15mm$$

But

$$2r = h$$

Therefore

$$2 \times 25.15mm = h$$

$$h \approx 50.3mm$$

The approximated dimensions needed to produce the optimum materials are $h \approx 50.3mm\ and\ r \approx 25.15mm$. In that case, let's compute the container's surface area using the optimized measurements. Then, as in the procedure below, compare the findings to the raw material's original pre-calculated area.

$$Cy_{SA} = 2\pi r^2 + 2\pi rhCy_{SA} = 2 \times \pi \times 25.15^2 + 2 \times \pi \times 50.3 \times 25.15$$

$$Cy_{SA} \approx 11927.57mm^2$$

Optimization

Based on the above calculation, the area of the materials required to produce a typical cylindrical container after optimization is approximately $11927.57mm^2$. By comparison, we find that $11927.57mm^2 < 94800mm^2$. Using the procedure below, let's determine the percentage change between the two areas.

$$\%\ change = \frac{94800mm^2 - 11927.57mm^2}{94800mm^2} \times 100\%\ change \approx 87.42\%$$

According to this computation, optimizing a cylindrical container would save around 87.42% on the container's cost and the materials utilized. Like in a conical-shaped container, the calculations in the case of cylindrical containers have limitations, primarily based on the exact shape of the container in question. For example, the container in Diagram 3 has ununiformed dimensions, which this calculation did not consider. Therefore, the assumption that the container is uniform is challenging when optimizing real-life cylindrical containers. Another limitation lies in the container's lid. This should have also been considered in the calculation, meaning that the results are not 100% accurate. However, for the sake of this exploration, these assumptions will hold since they will help drive the point of conducting this exploration.

A Cube Container

Six-sided figures like cubes may package nearly any other kind of goods. Its equal sides and angles make it easy to stack and store, ensuring efficient use of space. Additionally, its sturdy structure protects the contents inside, making it an ideal choice for shipping delicate items. The versatility of a cube allows it to be used in various industries, ranging from food packaging to electronics, making it a popular choice among manufacturers. For example, consider the real lie container below.

Diagram 4: An example of a real-life cubic container

Although it has no purpose, fast-food establishments can utilize the container above to package food items. So, what is the best material to make such a container, considering the raw material's initial surface area? Let's use the arithmetic calculation shown below to get a solution to this query. Assuming a perfect cube, the surface area (C_{SA}) is given as

$$C_{SA} = 2lw + 2hw + 2lh$$

However, each side must be equal since it is believed to be a perfect cube. As a result, the formula above is modified as follows ("Surface Area of Cube - Formula, Definition, Examples");

$$C_{SA} = 2l^2 + 2l^2 + 2l^2 = 6l^2$$

Let us make l the subject of the formula based on the above equation, as indicated below;

$$\frac{C_{SA} = 6l^2}{6} \sqrt{\frac{C_{SA}}{6}} = \sqrt{l^2}$$

$$Opt_l = \sqrt{\frac{SA_c}{6}}$$

Conversely, a cube's volume is expressed as

$$C_V = l \times w \times h$$

When a perfect cube is taken into account, the volume formula modifies as follows;

$$C_V = l^3$$

Now let us use substitution knowledge to substitute $l\, with\, l_o$in the volume equation as shown below;

$$C_V = \left(\sqrt{\frac{C_{SA}}{6}}\right)^3$$

However, the preliminary computation indicates that the 94800mm^2 raw material utilized in this study covers many fluid products. Thus, the following is the optimal surface area:

$$Opt_V = \left(\sqrt{\frac{C_{SA}}{6}}\right)^3$$

$$100000^{\frac{2}{3}} = \frac{Cu_{SA}}{6}$$

$$100000^{\frac{2}{3}} \times 6 = Cu_{SA} \approx 12926.61mm^2$$

Optimized Material

Based on optimized measurements, the materials needed to build a standard cubic container with a minimum one-liter capacity are estimated to be around $12926.\, squared\, after$optimization. By contrasting this value with the original value of the accessible raw material, we find that the optimized area $(12926.61mm^2 < 94800mm^2)$ is less than the non-optimized area. Let's calculate the percentage change between the two areas using the procedure below.

$$\%\, change = \frac{94800mm^2 - 12926.61mm^2}{94800mm^2} \times 100\%\, change \approx 86.36\%$$

According to this computation, a cubic container's optimization might save about 86.36% on the container's cost and the materials consumed. Again, the calculation up to this point did not consider the measurements of the lid of a real-life container. Instead, I assumed a perfect closed cubic container, which is not the case in a real-life scenario. In reality, the lid of a container may add additional height or width, potentially altering the overall volume. Additionally, the lid may have its thickness, further impacting the capacity of the container. Therefore, to accurately calculate the true volume of a real-life container, one must account for these factors and incorporate the measurements of the lid into the equation. Only then can we precisely estimate the container's capacity in practical situations.

Comparison of the Different Shapes Used in This Exploration

At this point, I have examined the optimal volume of four cylindrical, conical, and cubic forms. A different volume and SA might be obtained from the primary material for every form considered in this research. This component is crucial because it will determine the most effective form for cost optimization. A summary of the volume determined during this expedition may be seen in the table below.

	Cylinder	**Cone**	**Cubic**
Optimized Area	87.42%	35.04%	86.36%

According to the above data, a container with a cylindrical form optimizes the most material, while one with a conical shape optimizes the least. As a result, a cylindrical container would be the best answer if the container's shape is best for optimizing a given raw material to avoid wastage.

Conclusion and Limitation

This exploration aimed at utilizing calculus to optimize the raw materials needed to produce plastic containers to eliminate waste. The exploration began by determining the measurements of a cuboid, which formed the foundation of the volume that needed to be optimized to arrive at an optimum outcome. The calculations conducted during the exploration were remarkable, allowing me to analyze various shapes commonly used in everyday life for constructing containers and elucidate their mathematical connections. However, several limitations were encountered in the exploration process. For instance, assumptions were made, including adherence to regular measurements of the containers used in the calculations. Another assumption involved overlooking the measurements of the container lids, potentially impacting the reliability of the results. Additionally, the exploration failed to consider variations in the thickness of the container walls, which could have further compromised the precision of the measurements.

Despite these limitations, the study yielded informative results and laid the groundwork for future research on container size and computational aspects. These identified limitations create room for further development and investigation of the subject. This entails considering additional factors and ensuring the work remains applicable in real-world situations.

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